INTRODUCTION

Acoustic Frequency Spectrum

____Infrasonic_________|______Audible__________|____Ultrasonic_____

20-50 Hz | 15-20 kHz

Infrasonic:
- Seismic waves
- Atmospheric disturbances
- Military communications – low frequency for deep propagation

Audible:
- Speech and hearing
- Psychoacoustics, noise, etc.
- Physiological and psychological effects
- Entertainment – recording, production, and instruments

Ultrasonic:
- Sonar – submarine detection and communications
- Cavitation – cleaners and cell disruptors
- NDE and medical imaging – detect flaws, disease, & blood flow; microscopy
- Medical Imaging and Therapy – Disease diagnosis, blood flow, HIFU ablation, sonoporation
- SAW – communications, delay lines, filters, and correlators

In order to lay the framework for acoustics, let's first have a refresher course (or possibly completely new course) on complex arithmetic:

Definition: The imaginary unit *j* is defined by the relation $j^2 = -1$ (for engineers, physicist use *i*).

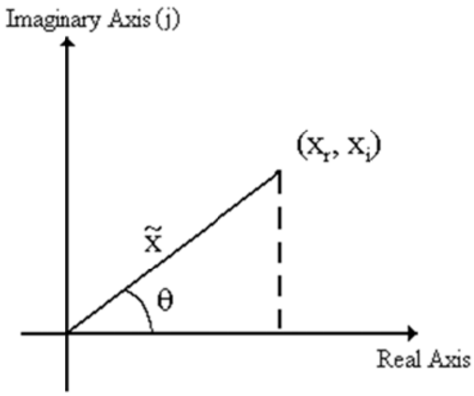
A complex number can be represented in <u>rectangular form</u> by $\tilde{x} = x_r + jx_i$ where x_r and x_i are real numbers.
The <u>magnitude</u> of a complex number $\tilde{x}$:

$$|\tilde{x}| = \sqrt{x_r^2 + x_i^2} = \sqrt{\tilde{x}\tilde{x}^*}$$

where the <u>complex conjugate</u> of $\tilde{x}$ is:

$$\tilde{x}^* = x_r - jx_i$$

We can represent complex numbers in a geometric (graphical) form:



Rectangular form: x_r , x_i

Polar Form: $|\hat{x}|, \theta$

Relations: $x_r = |\tilde{x}|\cos\theta$, $x_i = |\tilde{x}|\sin\theta$

$$|\tilde{x}| = \sqrt{x_r^2 + x_i^2} \;, \theta = \tan^{-1}\left(\frac{x_i}{x_r}\right)$$

Writing $\tilde{x}$ in polar form gives:

$$\tilde{x} = x_r + jx_i = |\tilde{x}|\cos\theta + j|\tilde{x}|\sin\theta$$

$$\tilde{x} = |\tilde{x}|\left(\cos\theta + j\sin\theta\right)$$

$\tilde{x} = |\tilde{x}|e^{j\theta}$ from Euler's identity (**memorize**): $e^{j\theta} = \cos\theta + j\sin\theta$

Mathematical operations:

Addition:
If $\tilde{x} = x_r + jx_i$ and $\tilde{y} = y_r + jy_i$ then $\tilde{x} + \tilde{y} = \left(x_r + y_r\right) + j\left(x_i + y_i\right)$
(This is much easier than using polar representation: $\tilde{x} + \tilde{y} = |\tilde{x}|e^{j\theta_x} + |\tilde{y}|e^{j\theta_y}$)

Multiplication: (much easier with polar form)

$$\tilde{x} \times \tilde{y} = x_r y_r + jx_i y_r + jx_r y_i - x_i y_i$$

$$\tilde{x} \times \tilde{y} = x_r y_r - x_i y_i + j\left(x_i y_r + x_r y_i\right) \quad \text{(Rectangular form)}$$

$$\tilde{x} \times \tilde{y} = |\tilde{x}|e^{j\theta_x} \times |\tilde{y}|e^{j\theta_y}$$

$$\tilde{x} \times \tilde{y} = |\tilde{x}||\tilde{y}|e^{j\left(\theta_x + \theta_y\right)} \quad \text{(Polar form)}$$

Division: (much easier with polar form)

$$\frac{\tilde{x}}{\tilde{y}} = \frac{x_r + jx_i}{y_r + jy_i} \times \frac{y_r - jy_i}{y_r - jy_i}$$

$$\frac{\tilde{x}}{\tilde{y}} = \frac{x_r y_r + x_i y_i + j\left(x_i y_r - x_r y_i\right)}{y_r^2 + y_i^2} \quad \text{(Rectangular form)}$$

$$\frac{\tilde{x}}{\tilde{y}} = \frac{|\tilde{x}|e^{j\theta_x}}{|\tilde{y}|e^{j\theta_y}}$$

$$\frac{\tilde{x}}{\tilde{y}} = \frac{|\tilde{x}|}{|\tilde{y}|} e^{j(\theta_x - \theta_y)}$$ (Polar form)

Powers:

$$\tilde{x}^{\gamma} = (x_r + jx_i)^{\gamma}$$

$$\tilde{x}^{\gamma} = \left[|\tilde{x}| e^{j\theta_x} \right]^{\gamma}$$

$$\tilde{x}^{\gamma} = |\tilde{x}|^{\gamma} e^{j\gamma\theta_x} = |\tilde{x}|^{\gamma} \left[\cos\gamma\theta + j\sin\gamma\theta \right]$$ (Much more revealing with polar form)

Example 1:

Let $\tilde{x} = 1 + i$, find $\tilde{x}^8 = (1+i)^8$

Sol:

Roots: Need to be especially careful with roots…

Example: Find $\sqrt[3]{8}$?

Sol: